1 Formula Sheet

Newtons Law of Cooling/Heating:

$$\frac{dT}{dt} = k(T - T_1)$$

Useful Trig Identities:

$$\begin{aligned}
\sin^2(x) + \cos^2(x) &= 1 \\
\tan^2(x) + 1 &= \sec^2(x) \\
\sin^2(x) &= \frac{1 - \cos(2x)}{2} \\
\cos^2(x) &= \frac{1 + \cos(2x)}{2} \\
\sin(2x) &= 2\sin(x)\cos(x) \\
\cos(2x) &= \cos^2(x) - \sin^2(x) \\
\sin(\cos^{-1}(x)) &= \sqrt{1 - x^2} \\
\cos(\sin^{-1}(x)) &= \sqrt{1 - x^2} \\
\sec(\tan^{-1}(x)) &= \sqrt{1 + x^2} \\
\tan(\sec^{-1}(x)) &= \begin{cases} \sqrt{x^2 - 1} & x \geq 1 \\ -\sqrt{x^2 - 1} & x \leq -1 \end{cases} \\
\sin(mx)\cos(nx) &= \frac{1}{2}[\sin(m+n)x + \sin(m-n)x] \\
\sin(mx)\sin(nx) &= -\frac{1}{2}[\cos(m+n)x - \cos(m-n)x] \\
\cos(mx)\cos(nx) &= \frac{1}{2}[\cos(m+n)x + \cos(m-n)x]
\end{aligned}$$

Useful Derivatives:

$$\begin{aligned}
D_x \sin^{-1}(x) &= \frac{1}{\sqrt{1 - x^2}} \quad -1 < x < 1 \\
D_x \cos^{-1}(x) &= -\frac{1}{\sqrt{1 - x^2}} \quad -1 < x < 1 \\
D_x \tan^{-1}(x) &= \frac{1}{1 + x^2} \\
D_x \sec^{-1}(x) &= \frac{1}{|x|\sqrt{x^2 - 1}} \quad |x| > 1
\end{aligned}$$

Some Useful Integral Forms:

$$\int \tan u du = -\ln|\cos(u)| + C$$
$$\int \cot(u) du = \ln|\sin(u)| + C$$
$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(\frac{u}{a}) + C$$
$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + C$$
$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}(\frac{u}{a}) + C$$

Series Formulas:

Taylor's Formula

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}$$

Hyperbolic Trig Definitions:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$

Note: The practice midterm may be a little longer than the actual midterm, but it reflects problems similar in difficulty to what you may encounter in the actual exam.

2 True/False

1. For an alternating series of the form $a_1 - a_2 + a_3 - a_4 + ...$ if $a_n > a_{n+1} > 0$ and $\lim_{n \to \infty} a_n = 0$, then the series converges.

2. A series that converges conditionally also converges absolutely.

3. A series converges absolutely in the interior of its convergence set.

4. You can find a Taylor series about any point x=a for any function f(x).

5. For an alternating series if the absolute ratio test give R=0, then the series converges conditionally.

3 Free Response

Using the test of your choice determine if the following sequences converge/diverge

1. $\sum_{n=1}^{\infty} \frac{3n-2}{n^2+3}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{2^n}$

Determine whether the following series conditionally converges, absolutely converges, or diverges.

3. $$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n^2}$

5. Find the Maclaurin series for $f(x) = \ln(1 + x)$

6. Find the convergence set and the radius of convergence for the Maclaurin series found in problem 5.

7. Using the Maclaurin series you found in problem 5 solve for the power series of $g(x) = \frac{1}{1+x}$

8. Find the 4th degree Taylor polynomial for $h(x) = \sinh(x)$ about the point a=1.